Circular Revisit Orbits Design for Responsive Mission Over A Single Target

Article *in* Acta Astronautica · June 2016

DOI: 10.1016/j.actaastro.2016.05.037

4 authors, including:

Taibo Li
Tsinghua University
4 PUBLICATIONS **14** CITATIONS

SEE PROFILE

Zhaokui Wang
Tsinghua University
66 PUBLICATIONS **240** CITATIONS

SEE PROFILE

Yulin Zhang□□□□□
Zhejiang University
106 PUBLICATIONS **534** CITATIONS

SEE PROFILE

Acta Astronautica 127 (2016) 219–225

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/aa

Circular revisit orbits design for responsive mission over a single target ☆

Taibo Li [a,*], Junhua Xiang [a], Zhaokui Wang [b], Yulin Zhang [b]

[a] College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China
[b] School of Aerospace, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 13 September 2015
Received in revised form
24 May 2016
Accepted 29 May 2016
Available online 4 June 2016

Keywords:
Circular revisit orbits
Repeat coverage orbits
Operationally response space
Optimal design
The minimum revisit time

ABSTRACT

The responsive orbits play a key role in addressing the mission of Operationally Responsive Space (ORS) because of their capabilities. These capabilities are usually focused on supporting specific targets as opposed to providing global coverage. One subtype of responsive orbits is repeat coverage orbit which is nearly circular in most remote sensing applications. This paper deals with a special kind of repeating ground track orbit, referred to as circular revisit orbit. Different from traditional repeat coverage orbits, a satellite on circular revisit orbit can visit a target site at both the ascending and descending stages in one revisit cycle. This typology of trajectory allows a halving of the traditional revisit time and does a favor to get useful information for responsive applications. However the previous reported numerical methods in some references often cost lots of computation or fail to obtain such orbits. To overcome this difficulty, an analytical method to determine the existence conditions of the solutions to revisit orbits is presented in this paper. To this end, the mathematical model of circular revisit orbit is established under the central gravity model and the J_2 perturbation. A constraint function of the circular revisit orbit is introduced, and the monotonicity of that function has been studied. The existent conditions and the number of such orbits are naturally worked out. Taking the launch cost into consideration, optimal design model of circular revisit orbit is established to achieve a best orbit which visits a target twice a day in the morning and in the afternoon respectively for several days. The result shows that it is effective to apply circular revisit orbits in responsive application such as reconnoiter of natural disaster.

1. Introduction

Responsiveness refers to the time between the development of a need and the subsequent solution that addresses that need. A key tenet of Operationally Responsive Space (ORS) is the ability to augment or reconstitute existing capabilities or to implement a new capability that is complimentary to fielded space assets [1,2]. Spacecraft flying in responsive orbits can be a significant asset to monitor natural disasters and to provide the necessary information to make informed decisions on the ground. For example, Russia's Resurs-P1 runs on a 475 km sun synchronous orbit with a six-day revisit period. The information obtained by it can be transmitted to a ground station in 12 h [3]. The orbits of The United States' “keyhole” series of satellites were proposed with perigees at about 315 km. These series of satellites can visit a particular area 1 or 2 times per day and conduct general and detailed survey over the target sites [4].

Traditionally, the performances of responsive imaging satellite in Low Earth Orbit (LEO) can be represented by the minimum revisit time, coverage percentage, resolution and so on [5]. Usually, low-altitude orbits may result in low coverage capability. This problem can be solved by making the orbits higher, but it will result in the loss of resolution. Increasing the number of the used satellite is another approach to solve this problem, which, however, increases the cost dramatically. Thus, it is important to optimize the responsive imaging orbits.

Literature [6] analyzed four different targets and different constellations of LEO spacecraft to understand their ability to not only support a single theater, but all four of the chosen theaters. The Nondominated Sorting Genetic Algorithm II (NSGA II) was used to design responsive orbits under the consideration of conflicting metrics including the orbital elements and launch programs of responsive vehicles [7]. In order to minimize the average revisit time (ART), the genetic algorithm was presented to find the Low-Earth fast access orbit of a single satellite and the orbits of each satellite in a constellation [8,9]. Genetic algorithms were also used to optimize the fuel consumption of constellations for responsive missions [10]. According to the ground track drift caused

☆ Supported by 863 Research Project of China (No. 2014AA7026081).
* Corresponding author.
E-mail address: nudt_ltb@163.com (T. Li).

http://dx.doi.org/10.1016/j.actaastro.2016.05.037
0094-5765/

by the non-spherical nature of the Earth, a strategy for the design and maintenance of a low-Earth repeat-ground track successive-coverage orbit was presented in [11]. However, all the researches mentioned above are achieved from the numerical calculation, which costs lots of computation by ignoring some possible analytical laws in theory.

As a design tool, it is preferable to a semi-analytic or analytic algorithm with the help of special periodic orbits. For remote sensing applications, many papers have focused their attention on the study of periodic orbits. The concept of studying repeat ground track orbits as true periodic orbits in an un-averaged model was first explored in [12]. Circi et al. [13] used repeat ground track orbits for designing and control for single and multi-plane satellite constellations. The results of a general study carried out on the Periodic Multi-Sun Synchronous Orbits (PMSSOs) in [14]. Such orbits allow cycles of observation of the same region in which the solar illumination regularly varies. In terms of elliptical orbit, a polynomial equation of PMSSO is generalized in [15]. Razoumny conducted researches in satellite constellation design for periodic coverage in [16], and the symmetrical and weakly symmetric satellite constellations are analyzed and compared in [16]. The analytic solutions for latitude coverage by single satellite and N-satellite arbitrary constellation were obtained in [17]. Based on the solutions, a new approach for satellite constellation design were proposed in [17]. Furthermore, a general method for minimization of the satellite swath width required under given constraint on the maximum revisit time was presented in [18]. All the research in [16–18] conducted on the satellites swath width and the maximum revisit time of recursive orbits. However, if we assume a point is visited only if it is on the nadir of the satellite, some different conclusion can be obtained. Therefore, a special type of recursive orbit was researched in this paper for the sake of responsiveness. The minimum revisit time instead of the maximum revisit time is considered.

Usually, the responsive satellite is proposed on nearly circular orbits and are low cost. To get a better performance with such satellites, special repeat coverage orbits called circular revisit orbits which can visit the target site at the ascending stage and at the descending stage alternately are discussed in literature [19]. Because the nadir of such satellite coincides with the target twice in a revisit cycle, it offers a low cost satellite a better observation and a higher resolution. However, the J_2 perturbation has not been taken into consideration in the previous paper [19]. Under the J_2 perturbation, not only the solutions of the circular revisit orbit but also the relation between circular revisit orbit altitude and the latitude of the target site were obtained by using a numerical approach [20]. Both of them proposed a mathematical model to define the circular revisit orbit, but they did not analyze the existent conditions of circular revisit orbits and oppose a theoretically method to get the best circular revisit orbit over a given target.

As the previous did, the study of revisit orbit in this paper is limited into circular orbits. Taking the J_2 perturbation into consideration, the mathematical model of circular revisit orbits is established in this paper. Based on the mathematical model, a constraint function related to orbit inclination is introduced to solve the circular revisit orbit. Then, the monotonicity of the constraint function has been studied, and a simple bisection algorithm is proposed to get the solution of that function. Furthermore, the existent conditions and the number of solutions have been obtained. Finally, analytical optimal solution has been achieved to minimize the minimum revisit time.

This paper is divided into five sections and the rest is organized as follows. The mathematical model of the circular revisit orbit is established in Section 2, which analyzes the revisit conditions under the central gravity model and the J_2 perturbation. Section 3 presents a method to solve the circular revisit orbit, focusing on the underlying physics and the analytical relations among the variables. An optimization model is developed in Section 4 to choose a low cost circular revisit orbit whose minimum revisit time can make the target be visited twice in the day time. In Section 5, we draw some conclusions and discuss future work.

2. Mathematical model of the revisit orbit

2.1. Basic concepts of the revisit orbit

Traditionally, satellites for responsive applications embark very simple remote sensor due to the limited cost. Considering the satellite swath width, a general method for optimizing the remote sensor required under given constraint on the maximum revisit time (MRT) was proposed in [18]. In order to visit a target without a side-sway of satellite and make full use of a satellite with a very small swath, we suppose that a satellite will visit the target site only if the target site is on the nadir of the satellite in this paper. If a satellite visits the target site at the ascending stage and at the descending stage alternately in a single cycle, we define it runs at a revisit orbit. As illustrated in Fig. 1, when the satellite is at point A, its bottom point B coincides with the target site, and due to the earth rotation, when it is at point A', its bottom point B'coincides with the target site again. Apparently, if the satellite runs at a regressive orbit, it will visit the target site at the ascending stage and descending stage alternately and repeatedly. Therefore, the revisit orbit should be regressive.

Given the longitude and latitude coordinates of a target site, three steps can be summarized to design a revisit orbit. To begin,

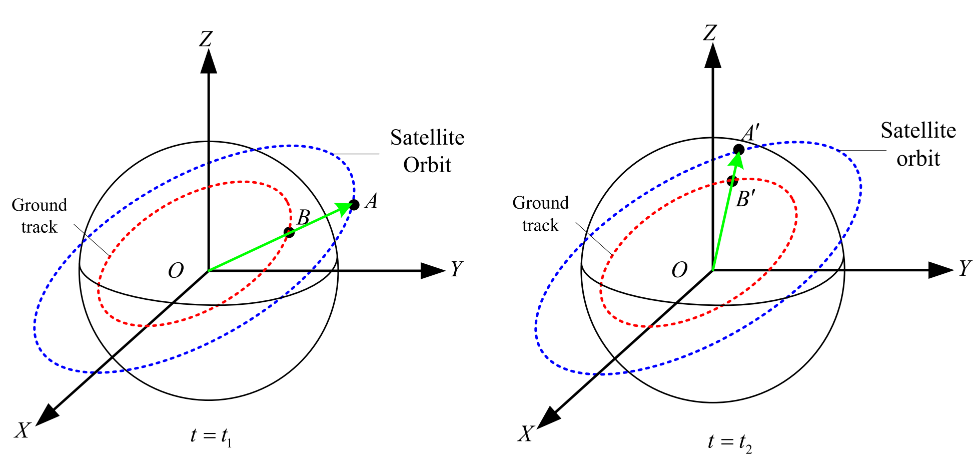

Fig. 1. A satellite which visits the target at ascending descending stages alternately.

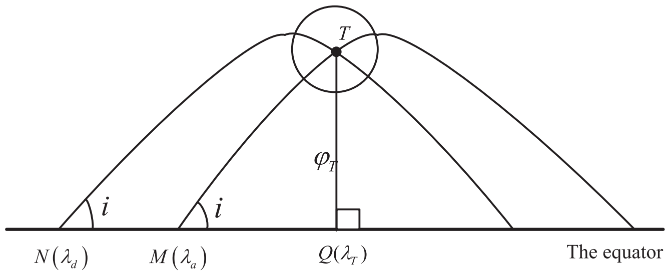

Fig. 2. Ground track of a revisit satellite.

the mathematical model of the revisit orbit is established and the constraint function of revisit orbit can be achieved. The orbital elements then can be solved through the function. A revisit orbit can be described by the orbital elements (a, e, i, Ω, ω), where a is the semi major axis, i is the orbit inclination, e is the eccentricity, Ω is the right ascension of ascending node, and w is the argument of perigee. Traditionally, the eccentricity of a responsive orbit is small, so circular orbits can be assumed, i.e., $e=0$. Accordingly, only three orbit elements a, i and Ω are needed to determine a revisit orbit.

2.2. Circular revisit orbit under the central gravity model

Given the target site T, let the longitude of the site be λ_T, and the latitude be ϕ_T. As shown in Fig. 2, the meridian of the target intersects with the equator at point Q. The ground track of a revisit satellite is also shown in Fig. 2. Point M represents the ascending node when the satellite is visiting the target site at the ascending stage. While it is at point N, when the satellite is visiting the target site at the descending stage. The ascending node of a revisit orbit passes point N and point M repeatedly and alternately.

According to the right spherical triangle $\widehat{MQT}$, the equations can be obtained as follows:

$$\sin\varphi_T = \sin\widehat{MT}\ \sin i \tag{1-a}$$

$$\sin\widehat{MQ} = \frac{\tan\varphi_T}{\tan i} \tag{1-b}$$

The Earth is assumed to be a perfect sphere under the central gravity model. Note that

$$\tan^{-1}\left(\frac{\cos i\sin\varphi_T}{\sqrt{\sin^2 i - \sin^2\varphi_T}}\right) = \sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) \tag{2}$$

Substituting Eq. (2) into Eq. (1), the longitudes of point M and N can be calculated as

$$\lambda_a = \lambda_T - \sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) + \frac{\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)}{2\pi} T\omega_e + \Omega_G \tag{3-a}$$

$$\lambda_d = \lambda_T - \pi + \sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) - \frac{\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)}{2\pi} T\omega_e + \frac{T\omega_e}{2}\Omega_G \tag{3-b}$$

where T is the orbital period, ω_e is the Earth spinning rate, Ω_G is the absolute longitude of Greenwich at the initial moment.

Set the right ascension of ascending node to be λ_a, then the satellite can visit the target at the first lap, that is to say,

$$\Omega = \lambda_T - \sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) + \frac{\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)}{2\pi} T\omega_e + \Omega_G \tag{4}$$

The constraint of the ascending node passing point N and point M alternately is

$$\lambda_a - \lambda_d = nT\omega_e \tag{5}$$

where n is an integer. Substituting Eq. (3) into Eq. (5), we can get

$$2\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right) + \left(\pi - 2\sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right)\right)\frac{2\pi}{T\omega_e} = (2n+1)\pi \tag{6}$$

Eq. (15) in [17] is similarity to Eq. (6). We can get Eq. (6) through substituting $\Delta\psi = n\pi$ to Eq.(15) in [17].It can be simplified as

$$\cos^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) = \left(\frac{n}{2} + \frac{\cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)}{2\pi}\right) T\omega_e \tag{7}$$

Note that the revisit orbit is regressive. If the regressive cycles are N, and the regressive period is D. The relationship between N and D can be described as [21]

$$\frac{2\pi}{T\omega_e} = \frac{N}{D} \tag{8}$$

where N and D are relatively prime integers. Substituting Eq. (8) into Eq. (7), we get the following equation

$$\cos^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) = \left(n\pi + \cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)\right)\frac{D}{N} \tag{9}$$

where n is an integer. Eq. (9) is consistent with the constraint equations in [19,20]. However the equation was achieved by studying the motion of the satellite bottom point in the two literatures. Instead, the equation is achieved through analyzing the conditions that the ascending node of circular revisit orbits must satisfy in this paper.

In summary, a circular revisit orbit can be described as

$$\frac{2\pi}{T\omega_e} = \frac{N}{D} \tag{10-a}$$

$$\cos^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) = \left(n\pi + \cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)\right)\frac{D}{N} \tag{10-b}$$

$$\Omega = \lambda_T - \sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right) + \frac{\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)}{2\pi} T\omega_e + \Omega_G \tag{10-c}$$

The revisit time for visiting the target at the ascending stage earlier before the descending stage can be calculated as

$$\Delta t_1 = nT_s + \frac{T}{\pi}\cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right) \tag{11}$$

While another revisit time can be obtained as

$$\Delta t_2 = (N-n)T_s - \frac{T}{\pi}\cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right) \tag{12}$$

By using Eq. (11) and Eq. (12) the average revisit time (ART) can be calculated as:

$$\Delta t_{ave} = \frac{1}{2}(\Delta t_1 + \Delta t_2) = \frac{N}{2}T_s \tag{13}$$

Apparently, the average revisit time is always half of the revisit period. Because the ability of responsiveness can be evaluated by the minimum revisit time, the performance of a revisit orbit can be rationally represented by

$$\Delta t_{min} = \min(\Delta t_1, \Delta t_2) \tag{14}$$

The satellite visits the target site twice per D days. If $D=1$, the satellite can visit the target site twice a day. The revisit time is presented as an alternate permutation of Δt_1, Δt_2.

2.3. Circular revisit orbit under the J_2 perturbation

In order to improve the accuracy of orbit computing, the J_2 perturbation should be taken into consideration. Circular revisit orbits are modulated because of the following three features under the J_2 perturbation:

(i) The orbit inclination and the semi-major axis are constant under the J_2 perturbation.
(ii) The orbital right ascension is varying. Accordingly, the Earth spinning rate should be replaced by the relative angular velocity ω between the Earth and the orbit plane.
(iii) The nodal period T_s is different from the orbital period T.

The rate of change of the orbital right ascension $\dot{\Omega}$ can be described as

$$\dot{\Omega}=-\frac{3}{2}J_2\left(\frac{R_e}{r}\right)^{\frac{7}{2}}\cos i\sqrt{\frac{\mu}{R_e^3}} \tag{15}$$

Then, the relative angular velocity between the Earth and the orbit plane can be calculated as $\omega=\omega_e-\dot{\Omega}$. The nodal period of a circular orbit can be described as [22]

$$T_s=T-\frac{3J_2}{8r^2}T(12-16\sin^2 i) \tag{16}$$

Substituting Eq. (15) and Eq. (16) into Eq. (10), circular revisit orbit under the J_2 perturbation can be described as

$$\frac{2\pi}{\left(\omega_e-\dot{\Omega}\right)T_s}=\frac{N}{D} \tag{17-a}$$

$$\cos^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right)=\left(n\pi+\cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)\right)\frac{D}{N} \tag{17-b}$$

$$\Omega=\lambda_T-\sin^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right)+\sin^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)\frac{D}{N}+\Omega_G \tag{17-c}$$

where D, N are relatively prime integers and n is an integer.

3. Solution of the circular revisit orbit

Only three orbit elements a, i and Ω are needed to design a circular revisit orbit. Due to Eq. (17), the RAAN, the semi-major axis and the orbit inclination are all functions of a trinity of D, N and n. Therefore the independent variables to design a circular revisit orbit are D, N and n. Given D, N and n, the existent conditions and the number of such orbits can be discussed in details.

3.1. Constraint function of the circular revisit orbit

It is clear that solutions' number of the inclination equals to the number of the circular revisit orbit when a trinity of D, N and n is given. Therefore solving the orbit inclination is a vital part of designing the revisit orbit.

The repetition factor q is introduced here to define N/D [13,14]. Traditionally, most responsive satellites are in LEO. Therefore, the region of q is to be set as $q\geq 1$ in this paper.

Let

$$F(i,n,q)=\cos^{-1}\left(\frac{\tan\varphi_T}{\tan i}\right)-\left(n\pi+\cos^{-1}\left(\frac{\sin\varphi_T}{\sin i}\right)\right)\frac{1}{q} \tag{18}$$

Apparently, a circular revisit orbit satisfies the equation $F=0$. In order to facilitate the description, $F(i,n,q)$ is represented by $F(i)$ for short below. The partial derivative for the orbit inclination of the constraint function $F(i)$ can be calculated as

$$\frac{\partial F}{\partial i}=\frac{\tan\varphi_T}{\sqrt{\tan^2 i-\tan^2\varphi_T}\sin i\cos i}-\frac{1}{q}\frac{\sin\varphi_T\cos i}{\sqrt{\sin^2 i-\sin^2\varphi_T}\sin i} \tag{19}$$

Because of that $q\geq 1$ for LEO satellites, we have

$$q^2\left(\cos^2\varphi_T-\cos^2 i\right)>\cos^2 i(\cos^2\varphi_T-\cos^2 i) \tag{20}$$

By expanding it, we further get the following equation

$$q^2\left(\cos^2\varphi_T-\cos^2 i\right)>\cos^2 i\cos^2\varphi_T\sin^2 i-\cos^4 i\sin^2\varphi_T \tag{21}$$

which equals to

$$q^2\left(\sin^2 i-\sin^2\varphi_T\right)>\cos^2\varphi_T\cos^4 i(\tan^2 i-\tan^2\varphi_T) \tag{22}$$

By using this result, we find that

$$G(i)\equiv\frac{q\sqrt{\sin^2 i-\sin^2\varphi_T}}{\sqrt{\tan^2 i-\tan^2\varphi_T}\cos^2 i\cos\varphi_T}>1,\ \forall\varphi_T\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \tag{23}$$

By multiplying Eq. (23) with $\sin\varphi_T$ and substituting the result into Eq. (19), one can conclude that

$$\begin{aligned}\text{I}:&\ \frac{\partial F}{\partial i}<0,\ \forall\varphi_T\in\left(-\frac{\pi}{2},0\right)\\ \text{II}:&\ \frac{\partial F}{\partial i}=0,\ \forall\varphi_T=0\\ \text{III}:&\ \frac{\partial F}{\partial i}>0,\ \forall\varphi_T\in\left(0,\frac{\pi}{2}\right)\end{aligned} \tag{24}$$

Nonlinear function $F(i)$ is a transcendental function, and there is no approach to obtain the analytical solution. However, due to the monotonicity of the function, dichotomy can be used to get the numerical solution. Besides, only one solution or none solutions of the inclination can be obtained if we substitute the given q and n into $F(i)$. Therefore, a trinity of D, $N$and n represents a circular revisit orbit or none orbits.

3.2. Relations among the inclination and the design parameters n and q

In order to find the solution of the orbit inclination with the given parameters q and n, this section focuses on the underlying physics and the analytical relations among the variables. The existent conditions and the number of solutions are achieved in three cases according to the targets located at (i) the equator ($\phi_T=0°$), (ii) the Northern Hemisphere ($0<\varphi_T<\pi/2$) and (iii) the Southern Hemisphere ($-\pi/2<\varphi_T<0$).

3.2.1. Target site located at the equator ($\phi_T=0°$)

Substituting $\phi_T=0°$ into Eq. (18) results in $F(i)=\pi/2-(n\pi+\pi/2)/q$. If $q=2n+1$, $F(i)$ has infinite number of solutions. In this case, the ground track of circular revisit orbit repeats in one day and the regressive cycles N is odd. Besides, the number of ground track crossovers at the equator is $2n+1$.

3.2.2. Target site located in the Northern Hemisphere ($0<\phi_T<\pi/2$)

When the target site is located in the Northern Hemisphere, $F(i)$ is a monotonically increasing function with the orbit inclination.

Table 1
Solutions of the orbit inclination in different cases in the Northern Hemisphere.

Case	n	$F(\varphi_T)$	$F(\pi/2)$	$F(\pi-\varphi_T)$	The solution of i
A	$n=0$	$=0$	>0	>0	φ_T
B	$0<n\le\frac{q}{2}-\frac{1}{2}+\frac{\varphi_T}{\pi}$	<0	≥0	>0	$i\in(\varphi_T,\frac{\pi}{2}]$
C	$\frac{q}{2}-\frac{1}{2}+\frac{\varphi_T}{\pi}<n<q$	<0	<0	>0	$i\in\left(\frac{\pi}{2},\pi-\varphi_T\right)$
D	$n=q$	<0	<0	$=0$	$\pi-\varphi_T$
E	$n>q$	<0	<0	<0	None

Substituting $i=\phi_T,\pi/2,\pi-\phi_T$ into Eq. (18), we have

$$F(\varphi_T)=-\frac{n\pi}{q},\quad F(\pi-\varphi_T)=\pi-\frac{n\pi}{q},\quad F\left(\frac{\pi}{2}\right)=\frac{\pi}{2}-\frac{1}{q}\left(n\pi+\frac{\pi}{2}-\varphi_T\right) \tag{25}$$

According to Eq. (25), the regions of solutions of the orbit inclination in different cases are listed in Table 1.

Due to the monotonicity of $F(i)$, there is only one solution of the orbit inclination in Case B and C. Case A/D corresponds to the case that the target site is located at the peak/nadir of the ground track respectively. Solutions in these two cases are rejected, because the orbit cannot revisit the target site at the ascending stage and descending stage alternately.

Let the latitude of the target site be 30° which is $\phi_T=30°$. The orbit inclination is plotted versus $1/q$ and n in Fig. 3. As shown in Fig. 3 when $n=2$, the solution exists only if $q>2$, and to make the solution of a prograde revisited orbit exists, $1/q$ must be not greater than 3/14. Fig. 3 also illustrates that the orbit inclination increases almost with $1/q$ linearly in most of the region, and increases in a gently trend at the end. Another feature to note is that the variation of orbit inclination caused by n is very small when $1/q$ is less than 0.1. The total changing trend is consistent with the theory prediction above.

3.2.3. Target site located in the Southern Hemisphere ($-\pi/2<\phi_T<0$)

When the target site is located in the Southern Hemisphere, $F(i)$ is a monotonically decreasing function with the orbit inclination. Substituting $i=-\phi_T,\pi/2,\pi+\phi_T$ into Eq. (18), we have

$$F(-\varphi_T)=\pi-\frac{(n+1)\pi}{q},\quad F(\pi+\varphi_T)=-\frac{(n+1)\pi}{q},\quad F\left(\frac{\pi}{2}\right)=\frac{\pi}{2}-\frac{1}{q}\left(n\pi+\frac{\pi}{2}-\varphi_T\right) \tag{26}$$

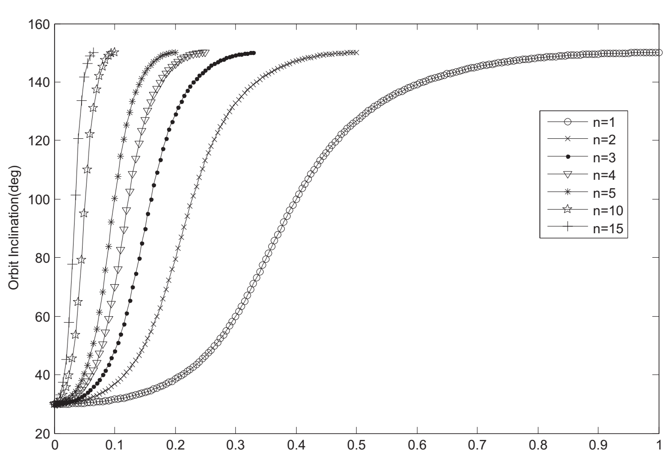

Fig. 3. The orbit inclination variation versus $1/q$ and n ($\phi_T=30°$).

The regions of the solutions of the orbit inclination in different cases are listed in Table.2. Similarly, there is only one solution of the orbit inclination in case A/B. Case C corresponds to the target site located at the peak or nadir of the ground track and it is also rejected.

Let the latitude of the target site be $-30°$ $\phi_T=-30°$. The orbit inclination is plotted versus $1/q$ and n in Fig. 4. From Fig. 4, we can see that, when $n=1$, $1/q$ must be smaller than 0.5 to make the solution exists. Furthermore, the solution of a prograde circular revisited orbit exists only in the case that $3/8\le1/q<1/2$. As well as Fig. 3, Fig. 4 also illustrates that the orbit inclination decreases almost with $1/q$ linearly in most of the region, and decreases in a gently trend at the end. Besides, the orbit inclination decreases with n in Fig. 4. The results here are also consistent with theory predictions.

In summary, the existent conditions and the number of solutions are listed in Table.3. Here, $[x]$ rounds toward the nearest integer which is not greater than x, and $[x]_*$ rounds toward the nearest integer lower than x. In case A, the target site is located at the equator and the target site is located in the Northern and Southern Hemisphere in case B and C respectively.

4. Optimal design model of circular revisit orbit

Given a target site defined by (ϕ_T, λ_T), several circular revisit orbits with different minimum revisit time can be obtained through solving $F(i)$. For a responsive mission of providing disaster relief, we expect that the minimum revisit time is as small as possible. That allows us get the current situation and make decision in time. However, for the sake of sustained observation for several days, it is meaningful to visit a target twice a day in the morning and in the afternoon respectively. By doing this, we can just use a MVIC to obtain enough information. In order to reach that goal, the minimum revisit time should be as close to 6 h as possible.

On the other hand, in terms of launch cost, not only a low circular revisit orbit height but also a small inclination are expected. However taking the atmospheric drag into consideration, the circular revisit orbit height should be higher than 150 km.

The optimal design model of circular revisit orbit is established in this paper to get a low launch-cost revisit orbit whose minimum time is close to 6 h. Recall that a trinity of D, N, and n represents a circular revisit orbit or none orbits, then D, N, and n are chosen to be the optimization variables .Furthermore, with the goal of covering a target twice a day, the value of D must be 1.Therefore only two optimization variables, i.e., N and n here. Overall, the optimization model is established as

$$\min_{i,n,q} J=w_1\left|\frac{\Delta t_{min}}{3600}-6\right|+w_2\frac{H}{R_e}+w_3\frac{i}{\pi/2}$$
$$\text{s. t.}\begin{cases}F(i,n,q)=0\\ \Delta t_{min}=\min(\Delta t_1,\Delta t_2)\\ H\ge150\text{ km}\\ 06:00\le t_1\le12:00\end{cases} \tag{27}$$

where $F(i,n,q)$ is defined as Eq. (18), $q=N/D$, Δt_1 and Δt_2 are defined as Eqs. (11) and (12). w_1, w_2 and w_3 are respectively the weights of minimum revisit time, orbit height and orbit inclination such that $w_1+w_2+w_3=1$. The variable t_1 is the local time when the target is firstly visited, and it is limited to be in the morning.

4.1. Design example

Set the value of w_1 to be 0.7, and both w_1 and w_2 to be 0.15. Orbit A in Table 4 is the optimal solution when the target site is

Table 2
Solutions of the orbit inclination in different cases in the Southern Hemisphere.

Case	n	$F(-\varphi_T)$	$F\left(\frac{\pi}{2}\right)$	$F(\pi+\varphi_T)$	The solution of i
A	$0 \le n < \frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi}$	>0	>0	<0	$i \in \left(\frac{\pi}{2}, \pi + \varphi_T\right)$
B	$\frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi} \le n < q-1$	>0	≤ 0	<0	$i \in \left(-\varphi_T, \frac{\pi}{2}\right]$
C	$n = q-1$	$=0$	<0	<0	$-\varphi_T$
D	$n > q-1$	<0	<0	<0	None

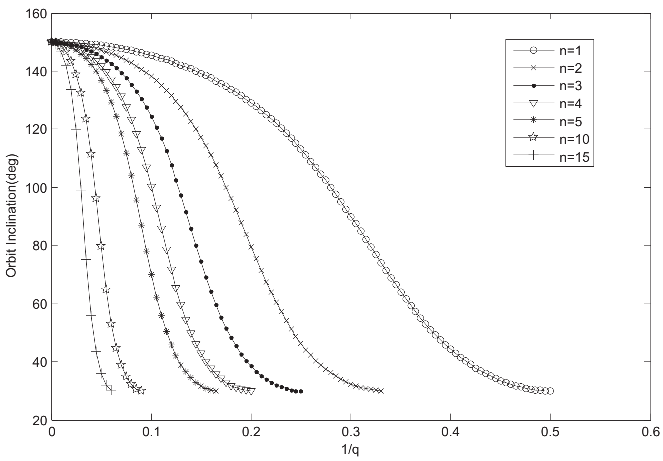

Fig. 4. The orbit inclination variation versus $1/q$ and n ($\phi_T = -30°$).

Table 3
Existent conditions and the number of solutions.

Case	The existent condition of the solution	The region of the solution	The number of solutions
A	None	$i \in [0, \pi]$	Infinite
B	$q > 1$	$i \in \left(\varphi_T, \frac{\pi}{2}\right]$	$\left[\frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi}\right]$
		$i \in \left(\frac{\pi}{2}, \pi - \varphi_T\right)$	$[q]_* - \left[\frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi}\right]$
C	$q > 2$	$i \in \left(-\varphi_T, \frac{\pi}{2}\right]$	$[q]_* - \left[\frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi}\right] - 1$
		$i \in \left(\frac{\pi}{2}, \pi + \varphi_T\right)$	$\left[\frac{q}{2} - \frac{1}{2} + \frac{\varphi T}{\pi}\right]_* + 1$

Table 4
Optimal solutions.

Orbit	h (km)	i (deg)	Ω (deg)	D	N	n	Δt_{min} (h)	J
A	188.5386	40.4883	106	1	16	4	6.1963	0.2093
B	219.3857	68.2617	91	1	16	11	6.1053	0.1926

defined as $\phi_T = 30°$, $\lambda_T = 120°$. Orbit B in Table.4 is the optimal solution when the target site is defined as $\phi_T = -60°$, $\lambda_T = 120°$.

4.2. Analysis of result

To test the above result, a numerical propagation of the orbits by using the above conditions is conducted. The scenario time is selected from 28 Oct 2015 00:00:00.000 UTCG to 7 Nov 2015 00:00:00.000 UTCG. Table 5 shows the access report of orbit A respective to target ($\phi_T = 30°$, $\lambda_T = 120°$). The access summary report of orbit B respective to target ($\phi_T = -60°$, $\lambda_T = 120°$) is in Table 6 .We can conclude that the optimal circular revisit orbit allows a satellite to visit the given target twice in the morning and in the afternoon alternately for about 10 days. Actually, it is 19 days for orbit B. On the other hand, this kind of orbit is low and prograde which means a low launch cost.

Overall speaking, the optimal circular revisit orbit is helpful for responsive application because of the following three features:

(i) The orbit inclination and the semi-major axis are small which means a low launch-cost.

Table 5
Access summary report from satellite to target by Orbit A.

Access	Start time (UTCG)	Stop time (UTCG)	Duration (s)
1	28 Oct 2015 11:32:52.867	28 Oct 2015 11:34:57.817	124.950
2	28 Oct 2015 17:44:37.029	28 Oct 2015 17:46:42.672	125.642
3	29 Oct 2015 11:02:15.056	29 Oct 2015 11:04:20.002	124.945
4	29 Oct 2015 17:13:59.197	29 Oct 2015 17:16:04.886	125.690
5	30 Oct 2015 10:31:37.237	30 Oct 2015 10:33:42.199	124.961
6	30 Oct 2015 16:43:21.373	30 Oct 2015 16:45:27.094	125.721
7	31 Oct 2015 10:00:59.409	31 Oct 2015 10:03:04.407	124.998
8	31 Oct 2015 16:12:43.558	31 Oct 2015 16:14:49.293	125.735
9	1 Nov 2015 09:30:21.571	1 Nov 2015 09:32:26.624	125.053
10	1 Nov 2015 15:42:05.751	1 Nov 2015 15:44:11.482	125.731
11	2 Nov 2015 08:59:43.722	2 Nov 2015 09:01:48.848	125.125
12	2 Nov 2015 15:11:27.950	2 Nov 2015 15:13:33.657	125.707
13	3 Nov 2015 08:29:05.863	3 Nov 2015 08:31:11.075	125.213
14	3 Nov 2015 14:40:50.154	3 Nov 2015 14:42:55.817	125.662
15	4 Nov 2015 07:58:27.992	4 Nov 2015 08:00:33.305	125.313
16	4 Nov 2015 14:10:12.363	4 Nov 2015 14:12:17.960	125.597
17	5 Nov 2015 07:27:50.110	5 Nov 2015 07:29:55.532	125.422
18	5 Nov 2015 13:39:34.576	5 Nov 2015 13:41:40.086	125.510
19	6 Nov 2015 06:57:12.217	6 Nov 2015 06:59:17.759	125.541
20	6 Nov 2015 13:08:56.790	6 Nov 2015 13:11:02.188	125.398

Table 6
Access summary report from satellite to target by Orbit B.

Access	Start time (UTCG)	Stop time (UTCG)	Duration (s)
1	28 Oct 2015 09:53:43.761	28 Oct 2015 09:55:06.133	82.373
2	28 Oct 2015 15:59:50.873	28 Oct 2015 16:01:24.514	93.642
3	29 Oct 2015 09:36:52.794	29 Oct 2015 09:38:15.197	82.402
4	29 Oct 2015 15:42:59.917	29 Oct 2015 15:44:33.574	93.657
5	30 Oct 2015 09:20:01.824	30 Oct 2015 09:21:24.265	82.441
6	30 Oct 2015 15:26:08.964	30 Oct 2015 15:27:42.631	93.666
7	31 Oct 2015 09:03:10.849	31 Oct 2015 09:04:33.337	82.487
8	31 Oct 2015 15:09:18.016	31 Oct 2015 15:10:51.684	93.668
9	1 Nov 2015 08:46:19.871	1 Nov 2015 08:47:42.414	82.543
10	1 Nov 2015 14:52:27.072	1 Nov 2015 14:54:00.734	93.662
11	2 Nov 2015 08:29:28.889	2 Nov 2015 08:30:51.495	82.606
12	2 Nov 2015 14:35:36.132	2 Nov 2015 14:37:09.782	82.606
13	3 Nov 2015 08:12:37.905	3 Nov 2015 08:14:00.582	93.630
14	3 Nov 2015 14:18:45.197	3 Nov 2015 14:20:18.827	93.630
15	4 Nov 2015 07:55:46.917	4 Nov 2015 07:57:09.674	82.756
16	4 Nov 2015 14:01:54.267	4 Nov 2015 14:03:27.870	93.603
17	5 Nov 2015 07:38:55.928	5 Nov 2015 07:40:18.771	82.843
18	5 Nov 2015 13:45:03.342	5 Nov 2015 13:46:36.912	93.570
19	6 Nov 2015 07:22:04.936	6 Nov 2015 07:23:27.873	82.937
20	6 Nov 2015 13:28:12.422	6 Nov 2015 13:29:45.952	93.529

(ii) The nadir of satellite can coincide with the target, thus the satellite does not need side-way to visit a target with a poor camera. Moreover, low orbit height offers a high resolution.

(iii) The revisit time is smaller than traditionally repeat ground track orbits. Furthermore, by choosing a suitable value of the minimum revisit time, we can apply circular revisit orbits to different responsive mission.

5. Conclusion

Special repeat coverage orbits called circular revisit orbits are proposed to find the best responsive orbit over a single target in this paper. The modeling and analysis of circular revisit orbits under the central gravity model and the J_2 perturbation are carried out. Constraint function of revisiting a particular target and the function of minimum revisit time are achieved. The existence conditions and the number of such orbits have been obtained through studying the monotonicity of the constraint function. Finally, the optimization model of circular revisit orbit is established. Based on the design model, we can achieve a low cost circular revisit orbit to visit a target twice a day in the morning and in the afternoon respectively. By doing this, we can just use a MVIC to obtain a sustained observation of a target for several days. Furthermore, for other responsive applications, different value of the minimum revisit time can be chosen to get a well performance. The conclusion in this paper has certain significance for engineering design a responsive orbit. Works in the future will be addressed in design of a responsive imaging constellation for the support of several target sites.

Acknowledgment

The authors are grateful to the anonymous reviewers for their helpful suggestions and remarks, and the helpful discussions by Dr. Xueyang Zhang et al. in National University of Defense Technology. The corresponding author also acknowledges the encouragement and care by Shigang Li (father), Hongling Yi (mother).

References

[1] J.R. Wertz, Coverage, responsiveness, and accessibility for various responsive orbits, in: Proceedings of the AIAA 3rd Responsive Space Conference, AIAA Paper RS3-2005-2001, Los Angeles, CA, Apr 25–28, 2005.
[2] J.G. Walker, Continuous whole-Earth coverage by circular-orbit satellite patterns. RAE Technical Report 77044, Mar 1977.
[3] Yaofeng Si, Kai Xu, A new series of remote sensing satellite called "Resurs-P1" have been launched by Russia, Space Int. (8) .
[4] Wenbo Jing, Yongming Gao, Zhihuan Wu, et al., Multi-software co-optimization of minisat orbit for observation mission, Comput. Simul. 32 (2) (2015).
[5] J.F. Carbary, C.I. Meng, Space remote sensing systems: an introduction, Eos Trans. Am. Geophys. Union 68 (42) (1987) 1131-1131.
[6] T.J. Mosher, K.E. Hamera, S.A. Cox. Responsive imaging constellations for support of geographically dispersed theaters, in: Proceedings of the AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Hawaii, 2008.
[7] F.U. Xiaofeng, Orbit design for responsive space using multiple-objective evolutionary computation, Chin. J. Space Sci. 32 (2) (2012) 238–244.
[8] R.D. Kim, O.C. Jung, H. Bang. A computational approach to reduce the revisit time using a genetic algorithm, in: Proceedings of the International Conference on Control, Automation and Systems, ICCAS'07, IEEE, 2007, pp. 184–189.
[9] H.D. Kim, H. Bang, O.C. Jung, et al. A Heuristic Approach to Design an Orbit for the Temporary Reconnaissance Mission Using a Few LEO Satellites, 2008.
[10] H.D. Kim, H. Bang, O.C. Jung, Genetic design of target orbits for a temporary reconnaissance mission, J. Spacecr. Rocket 46 (3) (2009) 725–728.
[11] X. Fu, M. Wu, Y. Tang, Design and maintenance of low-Earth repeat-ground-track successive-coverage orbits, J. Guid. Control. Dyn. 35 (2) (2012) 686–691.
[12] M. Lara, Searching for repeating ground track orbits: a systematic approach, J. Astronaut. Sci. 47 (3-4) (1999) 177–188.
[13] C. Circi, E. Ortore, F. Bunkheila, Satellite constellations in sliding ground track orbits, Aerosp. Sci. Technol. Vol. 39 (2014) 395–402.
[14] E. Ortore, C. Circi, F. Bunkheila, C. Ulivieri, Earth and Mars observation using periodic orbits, Adv. Space Res. 49 (1) (2012) 185–195.
[15] C. Circi, E. Ortore, F. Bunkheila, C. Ulivieri, Elliptical multi-sun-synchronous orbits for Mars exploration, Celest. Mech. Dyn. Astron. 114 (3) (2012) 215–227.
[16] Y.N. Razoumny, New research methodology for Earth periodic coverage and regularities in parametric localization of optimal low-Earth-orbit satellite constellations, Adv. Astronaut. Sci. 150 (2014) 3117–3136.
[17] Yury N. Razoumny. Analytic solutions for Earth discontinuous coverage and methods for analysis and synthesis of satellite orbits and constellations, in: Proceedings of the AIAA/AAS Astrodynamics Specialist Conference, 2014.
[18] Yu.N. Razoumny, Method for satellite orbit and constellation design for Earth discontinuous coverage with minimal satellite Swath under the given constraint on the maximum revisit time, Adv. Astronaut. Sci. (2015) 2337–2356.
[19] Jianfeng Fan, The design method of the orbit of satellite conducting repeated observation and continuous monitoring on specific targets, Chin. Space Sci. Technol. (4) .
[20] Wendun Zhang, The optimization design research on the constellation of small satellites covering the hotspots, Natl. Univ. Def. Technol. (2014) 48–51.
[21] M.J. Sidi, Spacecraft Dynamics and Control: A Practical Engineering Approach, Cambridge Univ. Press, New York, 1997.
[22] D. Vallado, Fundamentals of astrodynamics and applications, 3rd edition, Hypothesis Journal Com, 2007.